



ELSEVIER

Available online at www.sciencedirect.com

SCIENCE @ DIRECT®

Current Applied Physics 3 (2003) 449–455

Current
Applied
Physics

Physics, Chemistry and Materials Science

www.elsevier.com/locate/cap

An experimental study on the phase shift between piston and displacer in the Stirling cryocooler

S.J. Park ^{a,b,*}, Y.J. Hong ^a, H.B. Kim ^a, K.B. Lee ^b^a Thermal and Fluid Environmental Department, Korea Institute of Machinery and Materials, Daejeon 305-343, South Korea^b Department of Mechanical Engineering, Pusan National University, Pusan 609-735, South Korea

Received 4 June 2003; accepted 22 July 2003

Abstract

In the design of the split-type free displacer Stirling cryocooler, the motion of the displacer is very important to the cooling capacity, which depends on the working gas pressure, the swept volume in the compression space and the expansion space, the operating frequency, the phase shift between the piston and the displacer, and other factors. This study describes design and manufacture of the Stirling cryocooler actuated by the electric force of the dual linear motor. The cool-down characteristics of the cold end with laser displacement sensor in the expander of the Stirling cryocooler were evaluated. The charging pressure was 15 kgf/cm² and the operating frequency was 50 Hz. The input power and the lowest temperature were about 32 W and 67 K, respectively. The displacement of the piston was measured by LVDTs (linear variable differential transformers), the displacement of the displacer was measured by the laser optic method, and the phase shift between piston and displacer was investigated. As the peak-to-peak pressure of the compressor increased, the peak-to-peak displacement of the displacer increased. The peak-to-peak displacement of the displacer increased in the range of 0–64.5 Hz (the resonant frequency of the displacer), but decreased steeply when the operating frequency was higher than the resonant frequency. Finally, when the phase shift between the displacements of the piston and the displacer was 45°, the operating frequency was optimum and was decided by the resonant frequency of the expander, the mass and cross-sectional area of the displacer and the constants of friction and flow resistance.

© 2003 Elsevier B.V. All rights reserved.

PACS: 06.30.Gv; 07.20.Mc; 07.57.Ty

Keywords: Charging pressure; Operating frequency; Cooling capacity; Phase shift

1. Introduction

Over the past decade and a half there has been rapid development of Stirling cryocoolers, mainly for military and space applications. The cryocoolers working on the Stirling cycle are characterized by high efficiency, fast cool-down, small size, light weight, low power consumption and high reliability. Stirling cryocoolers have been widely used for the cooling of infrared sensors and high temperature superconducting filters to the temperature of the liquid nitrogen. Recently, the split-type Stirling cryocoolers driven by linear compressors have

been used to fit the requirements of a long operating life [1–3].

Free-piston Stirling engines acting as power systems were invented by William Beale in the early 1960s and have been in continuous development at Sunpower since that time. The first linear free-piston Stirling cryocooler was developed at the Philips Laboratories, Eindhoven, Haarhius (1978) [4]. Later De Jonge (1979) presented a classic paper related to the theoretical analysis of free-piston Stirling machines [5].

The FPF (free piston free displacer) Stirling cryocooler consists of two compressor pistons driven by linear motors, which make pressure waves, and a pneumatically driven displacer piston with a regenerator. In general, the efficiency of the Stirling cryocooler is mainly affected by the efficiency of the linear motor, the resonant frequency of the compressor and the expander,

* Corresponding author. Address: Thermal and Fluid Environmental Department, Korea Institute of Machinery and Materials, Daejeon 305-343, South Korea. Tel.: +82-42-868-7394; fax: +82-42-868-7335.

E-mail address: sjpark@kimm.re.kr (S.J. Park).

the displacement of the piston and the displacer, and the phase shift between the piston and the displacer.

The displacer in the expansion cylinder is free and oscillates simply as a result of the gas force acting upon it. The gas force is provided by the actuating piston oscillating in the compression cylinder with a phase shift relative to the displacer motion. The stroke variation of the displacer indicates the volume change in the expansion space, and its pressure–volume diagram illustrates the work done by the gas on the displacer. This positive work results in the refrigeration effect, and the cold finger becomes cold.

Therefore, in the design of the split-type free displacer Stirling cryocooler, the motion of the displacer is very important to the cooling capacity [6], and so the measurement of the displacer's displacement is indispensable to the development of the split-type Stirling cryocooler.

Yang et al. [7] proposed a relative displacement measurement method, which is non-contact by the use of the LVDT, to calibrate the static sensitivity of the measurement of the displacer motion inside the cold finger of a split-type Stirling cryocooler. Xiang et al. [8] took the displacer motion signal to identify the movement damping of the displacer, which changes along with the lowering of the refrigeration temperature, in a split-cycle free piston Stirling cryocooler. Zhang et al. [9] conducted an experimental investigation of the dynamic pressure distribution in a split-piston Stirling cryocooler system, where the displacer motion was measured with a Hall-effect displacement transducer.

In this study, for the given FPDF Stirling cryocooler, a laser displacement sensor was used to measure the displacement of the displacer, and the experimental results of the performance and optimum operating frequency of the Stirling cryocooler according to the variations of the phase shift between the piston and the displacer are presented.

2. Design and manufacturing of the Stirling cryocooler

Fig. 1 shows a schematic view of the FPDF Stirling cryocooler. The FPDF Stirling cryocooler consists of two major parts: the linear compressor module and the expander module.

The linear compressor consists of linear motors, inner and outer yokes, permanent magnets, coils, cylinders, and pistons and springs, while the expander module consists of a displacer, a regenerator in the displacer, a displacer cylinder, a spring and a heat exchanger.

The displacer with the regenerator is actuated by the pressure difference between the expansion space and the compression space. The equation of motion of the displacer is as follows [10]:

$$m\ddot{x} + c\dot{x} + kx = F, \quad (1a)$$

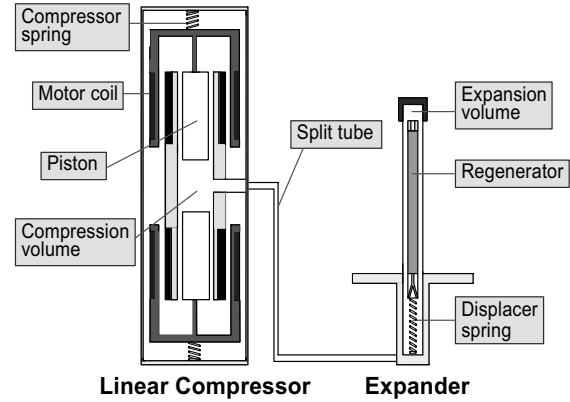


Fig. 1. A schematic diagram of the FPDF Stirling cryocooler.

$$f_n = 1/2\pi\sqrt{k/m}, \quad (1b)$$

$$\zeta = c/\sqrt{4mk} \quad (1c)$$

with m as the moving mass, c as the friction coefficient, k as the spring constant, F as the driving force, x as the displacement, f_n as the natural frequency, and ζ as the relative damping. The Stirling cryocooler operates in a cyclically steady state very close to sinusoidal.

Thus the instantaneous pressure and displacement are as follows:

$$P = P_0 \sin(\omega t), \quad (2a)$$

$$x = x_0 \sin(\omega t + \varphi) \quad (2b)$$

with P as the instantaneous pressure, P_0 as the amplitude of the pressure wave, x_0 as the amplitude of the displacement, ω as the operating angular velocity, and φ as the phase angle. The solution of Eq. (1a) are well known as the dynamic characteristics of the forced vibration system. The operating frequency, the natural frequency of the displacer and the relative damping have effects on the dynamics of the displacer.

The ideal refrigeration (PV power) of the Stirling cryocooler can be written as the following equation:

$$Q_{\text{ideal}} = \pi \cdot f \cdot P_0 \cdot A \cdot x_0 \cdot \sin \varphi \quad (3)$$

with A as the cross-sectional area of the expansion space, and f as the operating frequency. As shown in Eq. (3), the ideal refrigeration is greatly affected by the operating frequency, the amplitude of the pressure and the dynamics of the displacer. Radiation loss, conduction loss, etc. should be considered with the actual cooling capacity. If the temperatures at the cold end are the same, thermal losses including those of the radiation and conduction will be the same order of magnitude. And if the amplitudes of the pressure wave are the same at a given displacer, only the operating frequency and the dynamic characteristics of the displacer will have an impact on the cooling capacity.

Table 1
Dimensions and materials of the FPF Stirling cryocooler

Items	Materials	Dimensions (mm)
Piston	SUS 304	ϕ 12×20
Displacer	SUS 304	ϕ 5.8×70
Regenerator	SUS 304	# 325
Magnet	NdFeB	ϕ 35×20

Table 1 shows the dimensions and materials of the piston, displacer, regenerator and magnet in the Stirling cryocooler. Helium is used as the working fluid in the Stirling cryocooler cycle because of its ideal gas properties, its high thermal conductivity, and its high ratio of specific heats.

2.1. Experimental procedure

Fig. 2 shows a schematic diagram of the experimental apparatus of the FPF Stirling cryocooler. Piezo-electric dynamic pressure sensors were used to monitor the pressure oscillations at the outlet of the compressor and the buffers at each end. LVDTs were used at each end of the compressor for the displacement measurement of the pistons. A laser displacement sensor was used to measure the displacement of the displacer. A silicon-diode thermometer was attached to the cold head to measure the temperature of the cold end. After attaching those, the apparatus of the Stirling cryocooler, except for the components of the room temperature region, was connected to the vacuum flange. During the experiment, a vacuum chamber was connected to a high vacuum pump under a pressure of 10^{-5} Torr.

The high vacuum pump system consists of a rotary roughing pump, a turbo-molecular pump and vacuum gauges [11,12].

The following tests were undertaken for the experimental analysis of the phase shifts among the pressures of the compression space and expansion space, displacements of the piston and the displacer in the Stirling cryocooler:

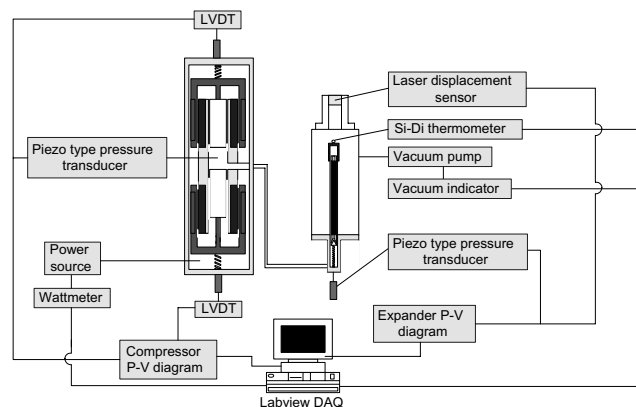


Fig. 2. Experimental apparatus of the FPF Stirling cryocooler.

1. The relationship between the output voltage of the laser displacement sensor and the position of the displacer.
2. The cool-down characteristics of the cold end with the laser displacement sensor in the expander of the Stirling cryocooler.
3. The amplitude of the displacer's stroke and phase shift under different temperatures.
4. The cooling capacity characteristics under the natural frequency of the displacer.
5. The displacer's amplitude under various charging pressures, the amplitude of pressure and of operating frequency.
6. The phase shifts between the displacements of the piston and the displacer under different operating frequencies.

3. Experimental results and discussion

The experimental setup for the calibration of the displacer's stroke, as shown in Fig. 3, consisted of the expander, vernier caliper, optical windows, laser displacement sensor, dewar and digital multimeter. The displacement of the displacer was measured by a caliper vernier with resolution of 0.01 mm, and the corresponding voltage output is recorded in the figure.

Fig. 4 shows that the relationship between the output voltage of the laser displacement sensor and the position of the displacer is almost linear.

Fig. 5 shows the cool-down characteristics of the cold end with laser displacement sensor in the expander of the Stirling cryocooler. The charging pressure was

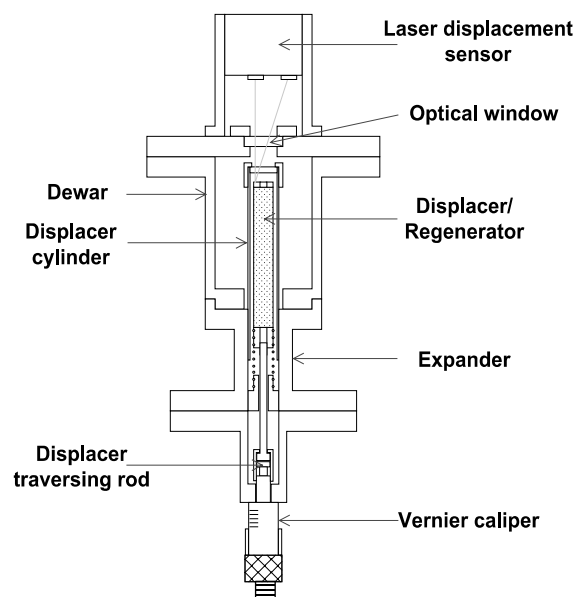


Fig. 3. The experimental setup for the calibration of the displacer's stroke.

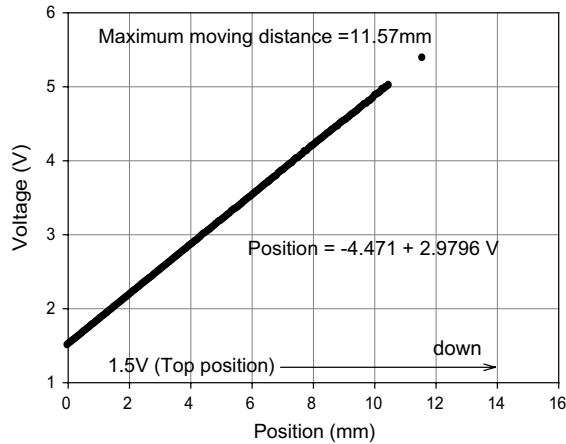


Fig. 4. Calibration of the laser displacement sensor.

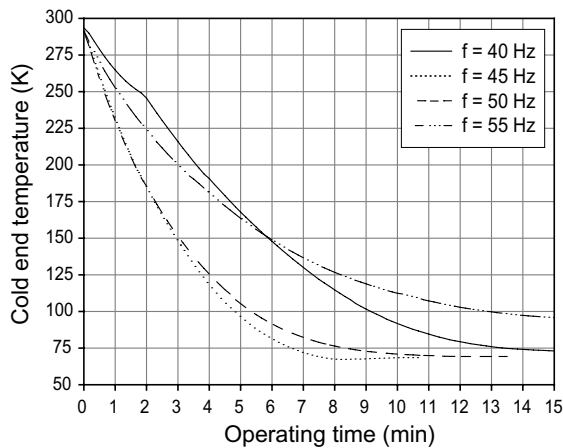


Fig. 5. Cool-down characteristics of the cold end.

15 kg_f/cm² and the operating frequency was 50 Hz. The input power and the applied voltage were about 32 W and 9 V, respectively. In this case, the lowest temperature was 67 K. Therefore, we were able to analyze the behaviors of the displacer under conditions below the liquid nitrogen temperature (77 K) for the cold finger of the expansion space.

Figs. 6 and 7 present the real time waves and phase shifts of the displacer motion measured by laser displacement sensor, piston motion, pressure, current and applied voltage.

Measured data for the peak-to-peak displacement of the displacer under different operating frequencies and for the peak-to-peak pressure of the compressor are presented in Fig. 8. The resonant frequencies of the compressor and expander are 54 and 64.5 Hz, respectively.

As the peak-to-peak pressure of the compressor increased, the peak-to-peak displacement of the displacer increased. The peak-to-peak displacement of the displacer increased in the range of 0–64.5 Hz, but decreased

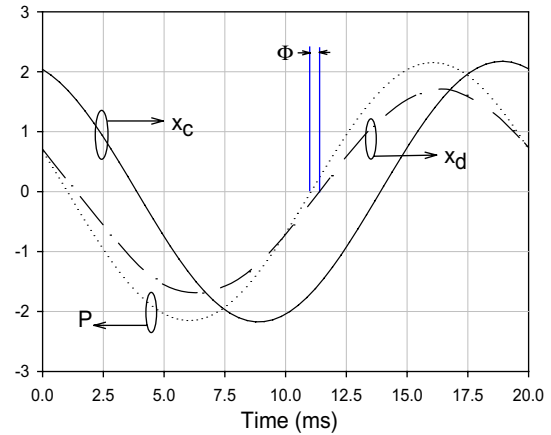


Fig. 6. Real time waves of the pressure, displacer motion, piston motion, current and applied voltage.

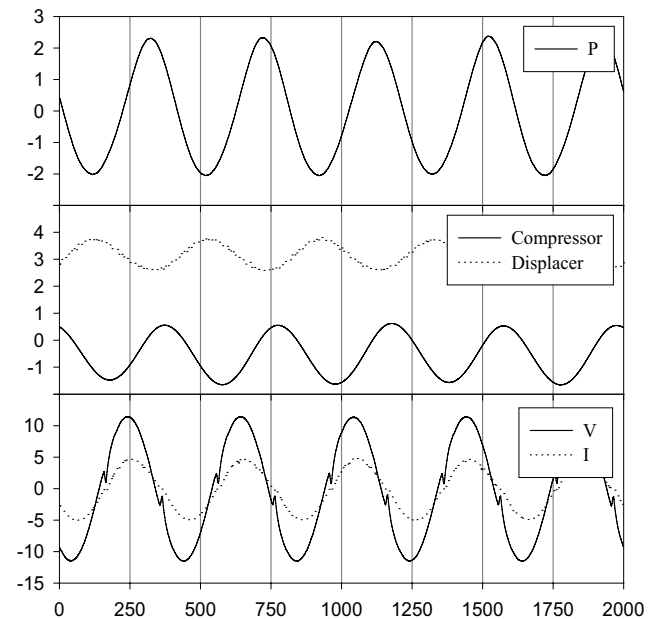


Fig. 7. Real time waves and phase shifts among the displacer motion, the piston motion and pressure.

steeply when the operating frequency was higher than the resonant frequency.

Fig. 9 shows the experimental results for normalized cooling capacity at 150 K under the different natural frequencies of the displacer (63.0, 67.4, 74.0 Hz). The natural frequency of the displacer could be adjusted by changing the mass or spring stiffness. In this study, the natural frequency of the displacer was adjusted by using springs of different stiffness but the same height. The maximum cooling capacity is occurred in the range of frequency ratio 0.75–0.85. Therefore, not only the resonance frequency of the linear compressor but also the natural frequency of the displacer should be considered in determining the operating frequency of the split-type

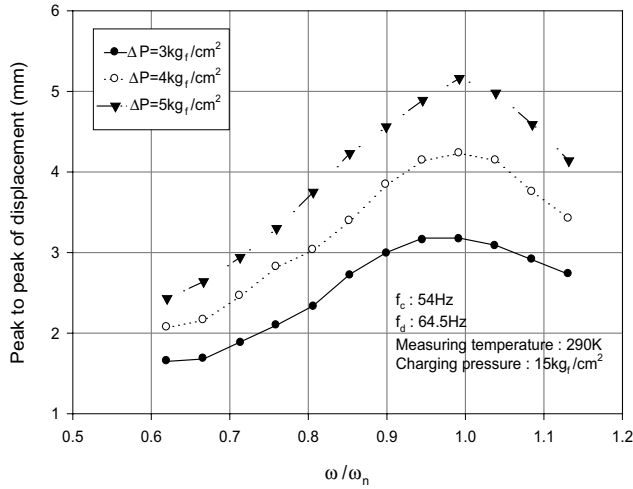


Fig. 8. Peak-to-peak displacement of the displacer under different operating frequency.

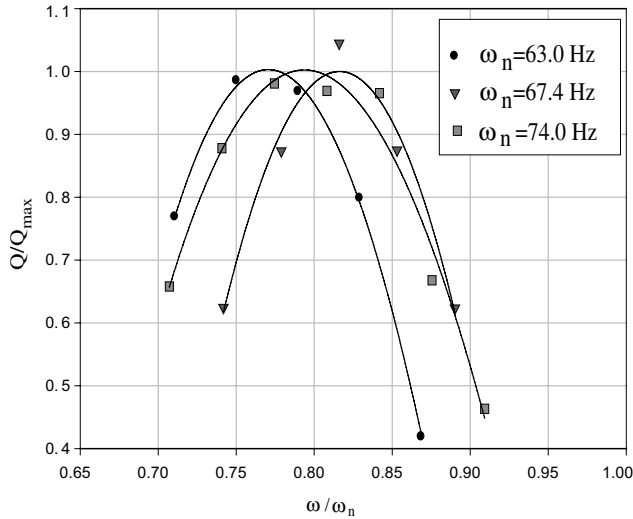


Fig. 9. The normalized cooling capacity characteristics with the different natural frequencies of the displacer.

free displacer Stirling cryocooler. For example, when the natural frequency of the displacer is in the range of 60–64 Hz, the optimum operating frequency of the split-type free displacer Stirling cryocooler is about 50 Hz.

Fig. 10 shows the displacer's amplitude ratio for normalized result at 40 Hz with frequency ratio to the natural frequency of the displacer and temperature of the cold finger. In this experiment, the pressure amplitude was constantly 3.5 kg_f/cm^2 .

The zero-to-peak stroke response x of the displacer driving system to the sinusoidal excitation at the frequency f is given by [13,14]

$$\frac{x}{x_0} = \frac{1}{\sqrt{[1 - (f/f_0)^2]^2 + [2 \cdot \zeta(f/f_0)]^2}} \quad (4)$$

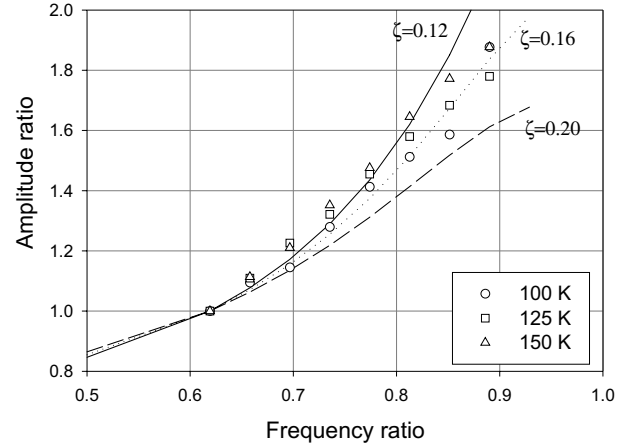


Fig. 10. Displacer's amplitude ratio and predicted damping with the frequency ratio and the temperature of the cold finger.

where $x_0 = F_d/k$ ($F = F_d \sin(2\pi ft)$) is the static displacement, $\zeta = C/C_c$ is the relative damping, $C_c = 2(km)^{1/2}$ is the critical damping coefficient, and $f_0 = (k/m)^{1/2}/(2\pi)$ is the natural frequency. The phase ψ of the stroke relative to the applied force is given by

$$\tan \psi = \frac{2\zeta(f/f_0)}{1 - (f/f_0)^2} \quad (5)$$

From Eq. (4), the relative damping is predicted as shown in Fig. 10.

As the temperature of the cold finger decreased, the relative damping increased. This may have resulted from the increase of the friction by the increase of the density and the viscosity of the helium gas.

Fig. 11 presents the effects of the amplitude of the pressure and of the operating frequency on the displacer's amplitude when the temperature of the cold finger is 150 K and the charging pressure is 20 kg_f/cm^2 .

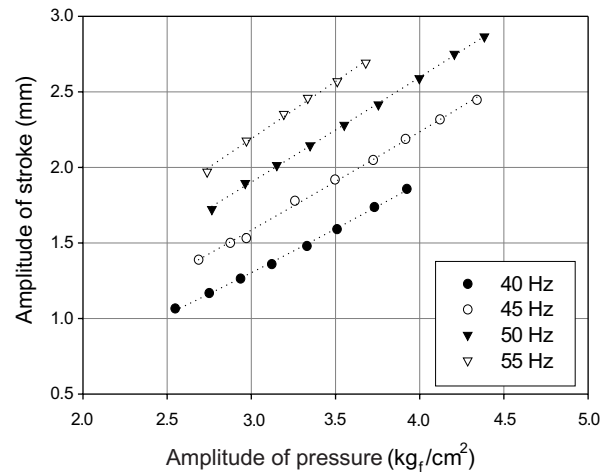


Fig. 11. Displacer's amplitude with the amplitude of the pressure and the operating frequency.

From Eq. (4) and Fig. 11, not only the relation between the displacer's amplitude and the driving force of the displacer but also the relation between the displacer's amplitude and the amplitude of the pressure in the compressor is linear. And, as the amplitude of the pressure in the compressor and the operating frequency increased, the displacer's amplitude increased.

The displacer's amplitudes for various charging pressures and operating frequencies having 3.5 kg/cm^2 of the constant pressure amplitude are shown in Fig. 12. The displacer's amplitude produced by the driving force of gas tends to increase as the charging pressure decreases and the operating frequency increases. Although the pressure amplitude is constant, if the charging pressure is different, the swept volume of the piston decreases, the flow rate and the pressure drop in the regenerator decrease, and finally, the driving force decreases. Therefore, the operating characteristics of the Stirling cryocooler are changed by the charging pressure. When the pressure amplitude is constant, there is little effect of the temperature in the cold finger on the displacer's amplitude.

Fig. 13 shows the experimental results for the amplitude of the displacer's stroke and the phase shift between displacements of the piston and displacer under different temperatures of the cold end and under the constant operating frequency of 50 Hz. The amplitude of the displacer's stroke and phase shift between displacements of the piston and the displacer decrease as the temperature of the cold end decreases.

Fig. 14 shows the phase shifts between displacements (ψ_{dc}) of the piston and the displacer under different operating frequencies. And Fig. 15 shows the phase shifts (ψ_{dp}) between displacements of the displacer and the pressure of the compression space under different operating frequencies. ψ_{dc} and ψ_{dp} decrease as the operating frequency increases and the temperature of the

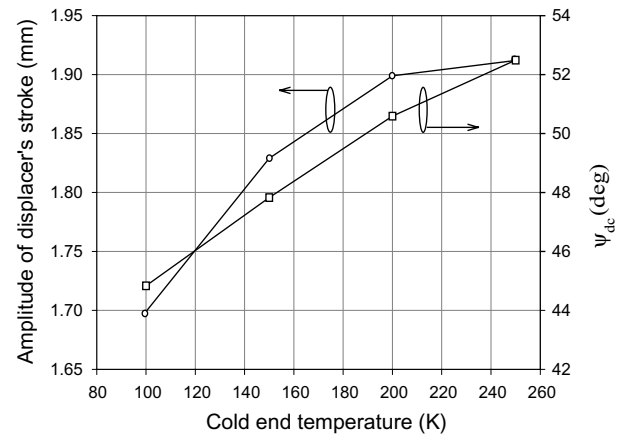


Fig. 13. Amplitude of the displacer's stroke and the phase shift with the different temperature.

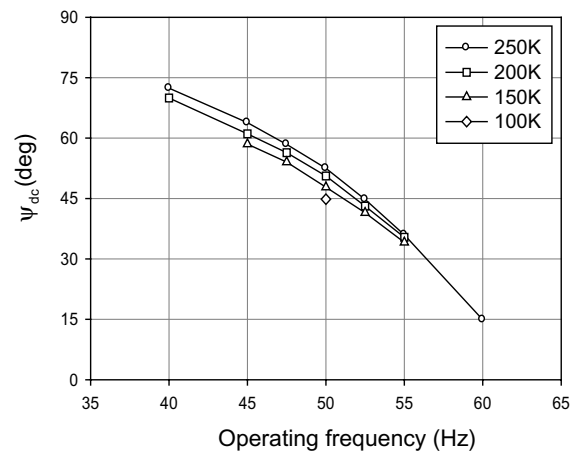


Fig. 14. Phase shifts between the displacements of the piston and the displacer with different operating frequencies.

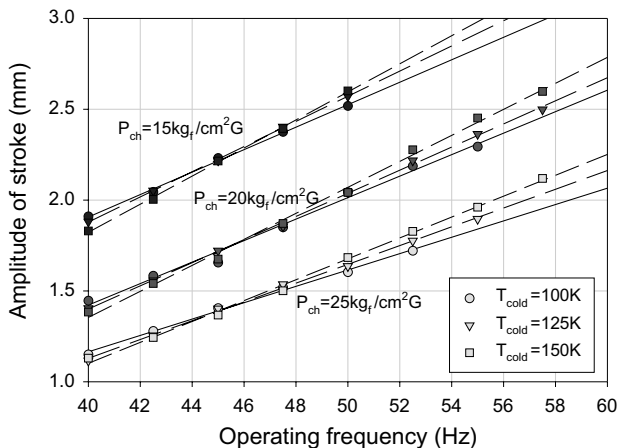


Fig. 12. Displacer's amplitudes for various charging pressures and operating frequencies.

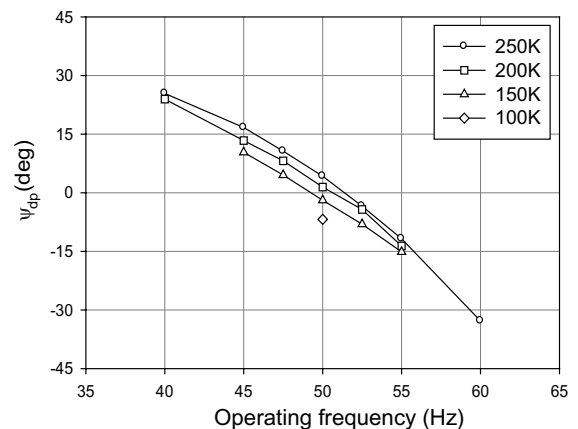


Fig. 15. Phase shifts between displacements of the displacer and the pressure of the compression space under different operating frequencies.

cold end decreases. As the temperature of the cold end decreases, ψ_{dc} and ψ_{dp} converge to 45° and -7° , respectively. And it is clear that ψ_{dc} and ψ_{dp} approach 0° and about -40° when the operating frequency coincides with the resonant frequency of the expander; in other words, refrigeration doesn't occur. Therefore, from the results of Figs. 14 and 15, when ψ_{dc} is 45° , operating frequency is optimum and is decided by the resonant frequency of the expander, the mass and cross-section area of the displacer and the constants of friction and flow resistance.

4. Conclusions

Stirling cryocooler actuated by the electric force of a dual linear motor was designed and manufactured. The displacement of the piston was measured by LVDTs, the displacement of the displacer was measured by the laser optic method, and the phase shift between the piston and the displacer was measured and evaluated by experiments.

The relationship between the output voltage of the laser displacement sensor and the position of the displacer was almost linear.

The cool-down characteristics of the cold end with laser displacement sensor in the expander of the Stirling cryocooler were investigated, and in this case, the charging pressure was 15 kgf/cm^2 and the operating frequency was 50 Hz. The input power and the lowest temperature were about 32 W and 67 K, respectively.

The maximum cooling capacity is occurred in the range of frequency ratio 0.75–0.85. Therefore, not only the resonance frequency of the linear compressor but also the natural frequency of the displacer should be considered in determining the operating frequency of the split-type free displacer Stirling cryocooler.

As the temperature of the cold finger decreased, the relative damping increased. This may have resulted from the increase of the friction by the increase of the density and viscosity of the helium gas.

Not only the relation between the displacer's amplitude and the driving force of the displacer but also the relation between the displacer's amplitude and the amplitude of the pressure in the compressor is linear. And, as the amplitude of the pressure in the compressor and the operating frequency are increased, the displacer's amplitude is increased.

When the pressure amplitude is constant, the displacer's amplitude produced by the driving force of the gas tends to increase as the charging pressure decreases and the operating frequency increases.

As the peak-to-peak pressure of the compressor increased, the peak-to-peak displacement of the displacer increased. The peak-to-peak displacement of the displacer increases in the range of 0–64.5 Hz (the resonant frequency of the displacer), but decreases steeply when the operating frequency is higher than the resonant frequency.

When the phase shift between the displacements of the piston and displacer is 45° , the operating frequency is optimum and is determined by the resonant frequency of the expander, the mass and cross-sectional area of the displacer and the constants of friction and flow resistance.

Acknowledgements

This work is supported by "Dual use technology program" with Wooyoung as industrial partners.

References

- [1] R. Radebaugh, Development of the pulse tube refrigerator as an efficient and reliable cryocooler, *Proceedings Institute of Refrigeration*, London, 2000.
- [2] S.J. Park et al., The effect of operating parameters in the Stirling cryocooler, *Cryogenics* 42 (2002) 419.
- [3] D.Y. Koh et al., A study on the linear compressor characteristics of the Stirling cryocooler, *Cryogenics* 42 (2002) 427.
- [4] G. Walker, *Miniature Refrigerators for Cryogenic Sensors and Cold Electronics*, Oxford University Press, New York, 1989.
- [5] A.K. De Jonge, A Small Free-Piston Stirling Refrigerator, *American Chemical Society*, 1979, p. 1139.
- [6] C.W. De Silva, *Control Sensors and Actuators*, Prentice Hall, Englewood Cliffs, NJ, 1989.
- [7] Y.P. Yang et al., New techniques for the non-contact measurement of displacer motion of a miniature split-Stirling cryocooler, *Cryogenics* 36 (8) (1996) 573.
- [8] Y. Xiang et al., Identification of the negative feedback relationship in split cycle free piston Stirling cryocooler system, *Cryogenics* 30 (September Suppl.) (1990) 216.
- [9] T. Zhang et al., Experimental investigation on the dynamic pressure distribution in a split piston Stirling cryocooler system, *Cryogenics* 30 (September Suppl.) (1990) 221.
- [10] R.F. Steidel Jr., *An Introduction to Mechanical Vibration*, Wiley, New York, 1979.
- [11] T. Fukuda et al., Development of stirling cryocooler in space, *Japanese Cryogenic Engineering* 27 (3) (1992) 252.
- [12] D. Verbeek et al., Performance of the signaal Usfa Stirling cooling engines, *International Cryocooler Conference* 7, 1993, pp. 728.
- [13] M.K. Heun et al., Investigation of gas effects on cryocooler resonance characteristics, *International Cryocooler Conference* 9, 1997, pp. 421.
- [14] W.T. Thomson, *Theory of Vibration with Applications*, third ed., Prentice Hall, Englewood Cliffs, NJ, 1988.